Converting between the misty and regular quantum formalisms

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These are some *very* brief notes to help a student learning regular quantum theory to connect the misty state formalism with the standard Hilbert space approach, as there are a couple of subtleties.

A misty state is just an unnormalized quantum state. For example $[W, W, -B]$ could be represented by a vector $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Doing so necessitates being careful to remember that the first entry corresponds to a white ball, the second to a black ball. That is, in a standard misty description the ordering of configuration doesn't matter, but once you start using vectors, which is the first step on the road to the regular quantum formalism, then ordering does matter.

I avoided normalizing vectors by introducing the "squaring rule" which, in this example, tells us that the probability of observing the ball to be white is $2^2/(2^2 + 1^2) = 4/5$. In standard quantum theory we would just normalize the state vector to

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} / \sqrt{5},$$

and the probabilities are then the squares of the vector entries.

The misty state formalism is just a different way of representing a subset of the quantum formalism which is actually universal for quantum computing (a fact first proven by Shih, references are at the end of the book). So in fact all quantum calculations could be done in this formalism to arbitrary accuracy. You may wonder where the complex amplitudes you get taught about are - they can be dealt with by having an extra ball in the mist. Effectively configurations with this extra ball white are "real parts of the quantum amplitudes" and those with the extra ball white are "imaginary parts of the quantum amplitudes".

Unitary evolution in the quantum formalism just gives the "boxes". For example the PETE box is just the Hadamard single qubit (ball) gate

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} / \sqrt{2}.$$

There is one subtlety to be careful about in going between the mist and the quantum state - in the misty formalism we can only incorporate boxes whose representation in the quantum formalism is via a unitary matrix which is proportional to a matrix of integer entries. For example $H \times \sqrt{2}$ contains only integers. It is a remarkable and nontrivial feature of quantum computation that such unitaries can be universal, ie used to simulate all unitaries, even ones with irrational complex entries.

Thus, while in principle possible, in practise doing all your computations within the misty formalism would incur some painful overhead (and be less compatible with math software packages!).